



GCE A LEVEL MARKING SCHEME

SUMMER 2023

**A LEVEL
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1**

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2023 MARK SCHEME

Qu.	Solution	Mark	Notes
1. a)	Domain of fg : $(0, \infty)$ Range of fg : $[1, \infty)$	B1 B1 (2)	
b)	$\cosh(x^2 - 1) = 3$ $x^2 - 1 = \cosh^{-1} 3$ $x^2 = 1 + \cosh^{-1} 3$ $x = \pm 1.662$ $x = 1.662$ since $x = -1.662$ not in domain	B1 M1 A1 (3) [5]	A0 for $x = \pm 1.662$ as final answer
2. a)	$\det \mathbf{A} = \pm \lambda(2\lambda - 16) \pm 1(-\lambda + 24) \pm 14(-2 + 6)$ $\det \mathbf{A} = \lambda(2\lambda - 16) - 1(-\lambda + 24) + 14(-2 + 6)$ $\det \mathbf{A} = 2\lambda^2 - 15\lambda + 32$	M1 A1 A1 (3)	Or equivalent rows/columns Fully simplified
b)	$\mathbf{A}$ is singular if $\det \mathbf{A} = 0$ METHOD 1: $2\lambda^2 - 15\lambda + 32 = 2\left(\lambda - \frac{15}{4}\right)^2 + \frac{31}{8}$ $2\left(\lambda - \frac{15}{4}\right)^2 + \frac{31}{8} > 0$ for all values of λ . Therefore $\mathbf{A}$ is non-singular. METHOD 2: When $2\lambda^2 - 15\lambda + 32 = 0$, $b^2 - 4ac = 225 - 256 = -31$ As $-31 < 0$ there are no real roots. Therefore $\mathbf{A}$ is non-singular. METHOD 3: When $2\lambda^2 - 15\lambda + 32 = 0$, $\lambda = \frac{15 \pm \sqrt{225 - 256}}{4}$ $\lambda = \frac{15 \pm \sqrt{31}}{4} i$ Therefore, there are no real roots. Therefore $\mathbf{A}$ is non-singular.	B1 M1 A1 B1 (M1) (A1) (B1) (M1) (A1) (B1) (4) [7]	si FT from (a) Accept $2\left(\lambda - \frac{15}{4}\right)^2 + \frac{31}{8} \neq 0$ for real values of λ

Qu.	Solution	Mark	Notes
3.a)	$z^n = \cos n\theta + i \sin n\theta$ $\frac{1}{z^n} = z^{-n} = \cos(-n\theta) + i \sin(-n\theta)$ $= \cos(n\theta) - i \sin(n\theta)$ $\therefore z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos(n\theta) - i \sin(n\theta)$ $= 2 \cos n\theta$	M1 A1 A1 (3)	1 use of de Moivre's theorem Both z^n and z^{-n} Convincing
b)	$\left(z + \frac{1}{z}\right)^6$ $= z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$ $= (z^6 + z^{-6}) + (6z^4 + 6z^{-4}) + (15z^2 + 15z^{-2}) + 20$ $= 2 \cos 6\theta + 12 \cos 4\theta + 30 \cos 2\theta + 20$ $\therefore (2 \cos \theta)^6 = 2 \cos 6\theta + 12 \cos 4\theta + 30 \cos 2\theta + 20$ $(2 \cos \theta)^6 = 64 \cos^6 \theta$ $32 \cos^6 \theta = \cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10$	M1 A2 m1 A1 A1 (6) [9]	Attempt to expand A1 at least 4 terms correct Condone unsimplified z terms si at least 2 pairs correct cao FT cao above
4.	METHOD 1: $\begin{pmatrix} 4 & -2 & 3 \\ 2 & -3 & 8 \\ 2 & 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$ By row operations: eg. $\begin{pmatrix} 1 & -3/2 & 4 \\ 0 & 4 & -13 \\ 0 & -1 & 17 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/2 \\ 10 \\ -19 \end{pmatrix}$ Then $\begin{pmatrix} 1 & -3/2 & 4 \\ 0 & -1 & 17 \\ 0 & 0 & 55 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/2 \\ -19 \\ -66 \end{pmatrix}$ Solving, $z = \frac{-6}{5} \quad y = \frac{-7}{5} \quad x = \frac{11}{5}$	M1 A1 A1 m1 A1	Attempt to reduce to echelon form (formal matrix notation not required) Reduction to $\begin{pmatrix} k & & \\ 0 & & \\ 0 & & \end{pmatrix}$ Reduction to $\begin{pmatrix} k & & \\ 0 & & \\ 0 & 0 & \end{pmatrix}$ cao

Qu.	Solution	Mark	Notes
	METHOD 2: Rearranging $x = -2y + \frac{1}{2}z$ and Substituting: 1st equation: $-10y + 5z = 8$ 2nd equation: $-7y + 9z = -1$ Solving, $z = \frac{-6}{5} \quad y = \frac{-7}{5} \quad x = \frac{11}{5}$	(M1) (A1) (A1) (m1) (A1)	Must be simplified Must be simplified cao
	METHOD 3: $\begin{pmatrix} 4 & -2 & 3 \\ 2 & -3 & 8 \\ 2 & 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$ By row operations: eg. $\begin{pmatrix} 1 & -3/2 & 4 \\ 0 & 4 & -13 \\ 0 & -1 & 17 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/2 \\ 10 \\ -19 \end{pmatrix}$ Then writing a pair of simultaneous equations in two variables e.g. $4y - 13z = 10$ $-y + 17z = -19$ Solving, $z = \frac{-6}{5} \quad y = \frac{-7}{5} \quad x = \frac{11}{5}$	(M1) (A1) (A1) (m1) (A1)	Attempt to reduce to echelon form (formal matrix notation not required) Reduction to $\begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$ Must be simplified cao
	METHOD 4: Attempt to calculate inverse matrix: $\frac{1}{-110} \begin{pmatrix} -29 & 10 & -7 \\ 18 & -10 & -26 \\ 14 & -20 & -8 \end{pmatrix}$ Multiplying matrices $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-110} \begin{pmatrix} -29 & 10 & -7 \\ 18 & -10 & -26 \\ 14 & -20 & -8 \end{pmatrix} \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{11}{5} \\ -\frac{7}{5} \\ -\frac{6}{5} \end{pmatrix}$	(M1) (A1) (A1) (m1) (A1) [5]	At least 5 entries correct in correct position (det not required) Fully correct (including det) M0 for answers without supporting working

Qu.	Solution	Mark	Notes
5. a)	METHOD 1: $\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!}$ $\sin 2x = 2x - \frac{8}{6}x^3 + \frac{32}{120}x^5 + \dots$ $\sin 2x = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \dots$ METHOD 2: $\sin 2x = \sin 0 + 2x \cos 0 + \frac{(2x)^2}{2!}(-\sin 0)$ $+ \frac{(2x)^3}{3!}(-\cos 0) + \frac{(2x)^4}{4!}\sin 0 + \frac{(2x)^5}{5!}\cos 0$ $\sin 2x = 2x - \frac{8}{6}x^3 + \frac{32}{120}x^5 + \dots$ $\sin 2x = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \dots$	M1 A1 (M1) (A1) (2)	Use of formula booklet expansion of $\sin x$ Use of formula booklet expansion of $f(x)$ with $\sin x$ and $2x$
b)	Differentiating, $2 \cos 2x = 2 - 4x^2 + \frac{4}{3}x^4$ $\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4$ $\cos 2x = 2 \cos^2 x - 1$ Therefore, $1 - 2x^2 + \frac{2}{3}x^4 = 2 \cos^2 x - 1$ $2 \cos^2 x = 2 - 2x^2 + \frac{2}{3}x^4$ $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4$	M1 A1 m1 A1 A1 (5) [7]	FT (a) oe cao

Qu.	Solution	Mark	Notes
6. a)	METHOD 1: $\tan \theta = \tan\left(\frac{\theta}{2} + \frac{\theta}{2}\right)$ $\tan\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \frac{\tan\left(\frac{\theta}{2}\right) + \tan\left(\frac{\theta}{2}\right)}{1 - \tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\theta}{2}\right)}$ $\tan\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \frac{t + t}{1 - t \times t}$ $\tan \theta = \frac{2t}{1 - t^2}$ METHOD 2: For $t = \tan\left(\frac{\theta}{2}\right)$, $\sin \theta = \frac{2t}{1+t^2} \text{ and } \cos \theta = \frac{1-t^2}{1+t^2}$ Therefore, $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \frac{2t}{1-t^2}$	E1 (E1) (1)	Convincing Convincing
b)	$x = r \cos(\theta) = \cos\left(\frac{\theta}{2}\right) \cos \theta$ $\frac{dx}{d\theta} = -\cos\left(\frac{\theta}{2}\right) \sin \theta - \frac{1}{2} \cos \theta \sin\left(\frac{\theta}{2}\right)$ When perpendicular to initial line: $\frac{dx}{d\theta} = -\cos\left(\frac{\theta}{2}\right) \sin \theta - \frac{1}{2} \cos \theta \sin\left(\frac{\theta}{2}\right) = 0$ $-\cos\left(\frac{\theta}{2}\right) \sin \theta = \frac{1}{2} \cos \theta \sin\left(\frac{\theta}{2}\right)$ $\tan(\theta) = -\frac{1}{2} \tan\left(\frac{\theta}{2}\right)$	M1 m1 A1 A1 (4)	Use of product rule $\frac{dx}{d\theta} = 0$ Convincing

Qu.	Solution	Mark	Notes
6. c)	$\frac{2t}{1-t^2} = -\frac{1}{2}t$ $2t = -\frac{1}{2}t + \frac{1}{2}t^3$ $4t = -t + t^3$ $t^3 - 5t = 0$ $t(t^2 - 5) = 0$ $t = 0, \quad t = \pm\sqrt{5}$ $\tan\left(\frac{\theta}{2}\right) = 0 \quad \text{or} \quad \tan\left(\frac{\theta}{2}\right) = \pm\sqrt{5}$ $\frac{\theta}{2} = 0 \quad \frac{\theta}{2} = 1.15 \dots \quad \frac{\theta}{2} = -1.15 \dots$ $\theta = 0 \quad r = 1$ $\theta = 2.30 \quad r = 0.408$ $\theta = -2.30 \quad r = 0.408$ $(1, 0), (0.408, 2.30), (0.408, -2.30)$	M1 A1 A1 A1 B2 (6)	Use of compound-angle Accept use of $\tan\left(\frac{\theta}{2}\right)$ in place of t throughout For at least 2 solutions Ignore angles outside range FT their $\frac{\theta}{2}$ values provided $r > 0$ B2 All correct, no additional sets B1 for 2 correct sets of coordinates
d)	$\text{Area} = \frac{1}{2} \int_0^\pi \cos^2\left(\frac{\theta}{2}\right) d\theta$ $= \frac{1}{2} \int_0^\pi \frac{\cos \theta + 1}{2} d\theta$ $= \frac{1}{4} \int_0^\pi (\cos \theta + 1) d\theta$ $= \frac{1}{4} [\sin \theta + \theta]_0^\pi$ $= \frac{1}{4} [(0 + \pi) - (0 + 0)]$ $= \frac{\pi}{4}$	M1 M1 A1 A1 A1 (5) [16]	Use of double-angle formula All correct, including limits

Qu.	Solution	Mark	Notes
7.	$ z = \sqrt{11^2 + (-2)^2} = 5\sqrt{5}$ $\arg(z) = \tan^{-1}\left(-\frac{2}{11}\right) = -0.179(85 \dots)$ Therefore cube roots: 1st root: $\sqrt[3]{5\sqrt{5}}(\cos -0.05995 + i \sin -0.05995)$ $= 2.232 - 0.134i$ 2nd root: $\sqrt[3]{5\sqrt{5}}(\cos 2.0344 + i \sin 2.0344)$ $= -1.000 + 2.000i$ 3rd root: $\sqrt[3]{5\sqrt{5}}(\cos 4.1288 + i \sin 4.1288)$ $= -1.232 - 1.866i$	B1 B1 M1 A1 M1 A1 A1 [7]	si si Use of de Moivre. FT $ z $ and $\arg(z)$ $+\frac{2\pi}{3}$ Note: $\sqrt[3]{5\sqrt{5}} = \sqrt{5}$
8. a)	Mean value $= \frac{1}{2-0} \int_0^2 \left(\frac{1}{\sqrt{x^2+4x+3}} \right) dx$ $\frac{1}{2} \int_0^2 \left(\frac{1}{\sqrt{(x+2)^2 - 1}} \right) dx$ $= \frac{1}{2} \left[\cosh^{-1} \left(\frac{x+2}{1} \right) \right]_0^2$ $= 0.373$	M1 m1 A1 A1 A1 (5)	Condone omission of $1/(b-a)$ Completing the square Must include $1/(b-a)$ or $\frac{1}{2} \ln(x+2+\sqrt{((x+2)^2-1)})$ or $\frac{1}{2} \ln(x+2+\sqrt{x^2+4x+3})$ Correct integration of $\frac{1}{\sqrt{x^2 \pm a^2}}$ cao No marks awarded for answer only.
b)	METHOD 1: Volume $= \pi \int_0^2 \left(\frac{1}{\sqrt{x^2+4x+3}} \right)^2 dx$ $= \pi \int_0^2 \frac{1}{x^2+4x+3} dx$ $= \pi \int_0^2 \frac{1}{(x+2)^2 - 1} dx$ $= \frac{\pi}{2} \left[\ln \left \frac{x+2-1}{x+2+1} \right \right]_0^2$ $= \frac{\pi}{2} \left[\ln \left \frac{x+1}{x+3} \right \right]_0^2$ $= \frac{\pi}{2} \left[\ln \frac{3}{5} - \ln \frac{1}{3} \right]$ $= \frac{\pi}{2} \ln \frac{9}{5}$	M1 A1 m1 A1 m1 A1	Condone omission of π Fully correct Must include π and limits Limits not required Correct integration Correct use of limits oe Must be exact Mark final answer

Qu.	Solution	Mark	Notes
8. b)	METHOD 2: $\left(\frac{1}{\sqrt{x^2 + 4x + 3}}\right)^2 = \frac{1}{x^2 + 4x + 3}$ $\frac{1}{x^2 + 4x + 3} = \frac{A}{x + 1} + \frac{B}{x + 3}$ $1 = A(x + 3) + B(x + 1)$ When $x = -1$, $1 = 2A$ $\therefore A = \frac{1}{2}$ When $x = -3$, $1 = -2B$ $\therefore B = -\frac{1}{2}$ $\text{Volume} = \pi \int_0^2 \left(\frac{1}{\sqrt{x^2 + 4x + 3}}\right)^2 dx$ $= \pi \int_0^2 \left(\frac{1}{2(x+1)} - \frac{1}{2(x+3)}\right) dx$ $= \pi \left[\frac{1}{2} \ln x+1 - \frac{1}{2} \ln x+3 \right]_0^2$ $= \pi \left[\left(\frac{1}{2} \ln 3 - \frac{1}{2} \ln 5 \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{2} \ln 3 \right) \right]$ $= \frac{\pi}{2} \ln \frac{9}{5}$	(M1) (A1) (M1) (A1) (m1) (A1) (6) [11]	Use of partial fractions Both values Condone omission of π Correct integration. Must include π Use of limits oe Must be exact Mark final answer

Qu.	Solution	Mark	Notes
9.	Dividing both sides by $(x + 1)$: $\frac{dy}{dx} + \frac{5y}{x+1} = x + 1$ Integrating factor: $e^{\int \frac{5}{x+1} dx}$ $= e^{5 \ln(x+1)} = (x + 1)^5$ Multiplying both sides: $(x + 1)^5 \frac{dy}{dx} + 5y(x + 1)^4 = (x + 1)^6$ Integrating: $(x + 1)^5 y = \frac{(x + 1)^7}{7} + c$ Substituting $32 \times \frac{1}{4} = \frac{128}{7} + c \rightarrow c = \frac{-72}{7}$ Solution $(x + 1)^5 y = \frac{(x + 1)^7}{7} - \frac{72}{7}$ When $x = 0$, $y = \frac{1}{7} - \frac{72}{7}$ $\rightarrow y = -\frac{71}{7} (-10.142857)$	M1 m1 A1 M1 m1 A1 B1 B1 [8]	cao Or equivalent, cao

Qu.	Solution	Mark	Notes
10.a)	METHOD 1: Writing $y = \sin^{-1}(2x + 5)$ $\therefore \sin y = 2x + 5$ $\frac{1}{2}(\sin y - 5) = x$ Differentiating, $\frac{dx}{dy} = \frac{1}{2} \cos y$ Since, $\sin^2 y + \cos^2 y = 1$ $(2x + 5)^2 + \cos^2 y = 1$ $\cos^2 y = 1 - (2x + 5)^2$ Therefore, $\frac{dx}{dy} = (\pm) \frac{1}{2} \sqrt{1 - (2x + 5)^2}$ $\frac{dy}{dx} = (\pm) \frac{2}{\sqrt{1 - (2x + 5)^2}}$ $\frac{dy}{dx} = \frac{2}{\sqrt{1 - (2x + 5)^2}}$ AND valid justification, eg. reference to the gradient of the graph. METHOD 2: Writing $y = \sin^{-1}(2x + 5)$ $\therefore \sin y = 2x + 5$ Differentiating, $\cos y \frac{dy}{dx} = 2$ $\frac{dy}{dx} = \frac{2}{\cos y}$ Since, $\sin^2 y + \cos^2 y = 1$ $(2x + 5)^2 + \cos^2 y = 1$ $\cos^2 y = 1 - (2x + 5)^2$ Therefore, $\frac{dy}{dx} = (\pm) \frac{2}{\sqrt{1 - (2x + 5)^2}}$ $\frac{dy}{dx} = \frac{2}{\sqrt{1 - (2x + 5)^2}}$ AND valid justification, eg. reference to the gradient of the graph.	M1 A1 B1 A1 A1 (M1) (A1) (B1) (A1) (A1) (5)	Rearrange and attempt to differentiate si Attempt to differentiate si

Qu.	Solution	Mark	Notes
10.b)	$1 - (2x + 5)^2 = -4x^2 - 20x - 24$ METHOD 1: When $-4x^2 - 20x - 24 = 0$ $x = -2$ or $x = -3$ Therefore the range of valid values is $-3 < x < -2$ METHOD 2: Valid when $1 - (2x + 5)^2 > 0$ $-4x^2 - 20x - 24 > 0$ $4x^2 + 20x + 24 < 0$ $4(x + 3)(x + 2) < 0$ Therefore, $-3 < x < -2$ METHOD 3: $\sin y = 2x + 5$ Valid when $-1 \leq 2x + 5 \leq 1$ $-3 \leq x \leq -2$ Therefore (due to fraction), $-3 < x < -2$	M1 A1 A1 (M1) (A1) (A1) (M1) (A1) (A1) (3) [8]	

Qu.	Solution	Mark	Notes
11.b)	$\frac{5}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$ $5 = A(x^2+9) + (Bx+C)(x-1)$ When $x = 1, 5 = 10A$ $\therefore A = \frac{1}{2}$ When $x = 0, 5 = 9A - C$ $\therefore C = -\frac{1}{2}$ Co-efficients of $x^2: 0 = A + B$ $\therefore B = -\frac{1}{2}$ $\int_{1.5}^3 \frac{5}{(x-1)(x^2+9)} dx$ $\int_{1.5}^3 \left(\frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}x - \frac{1}{2}}{x^2+9} \right) dx$ $= \int_{1.5}^3 \left(\frac{1}{2(x-1)} - \frac{x}{2(x^2+9)} - \frac{1}{2(x^2+9)} \right) dx$ $= \left[\frac{1}{2} \ln x-1 - \frac{1}{4} \ln x^2+9 - \frac{1}{6} \tan^{-1} \frac{x}{3} \right]_{1.5}^3$ $= 0.522 \text{ (0211808 ...)}$	M1 A1 A2 M1 A1 A2 A1 (9) [14]	A1 any 2 coefficients FT their A, B, C provided at least two non-zero values Three terms Three terms all correct (A1 any 1 term) cao No marks awarded for answers only.

Qu.	Solution	Mark	Notes
12.	$2 \cos \frac{4\theta + 2\theta}{2} \cos \frac{4\theta - 2\theta}{2} = \cos \theta$ $2 \cos 3\theta \cos \theta - \cos \theta = 0$ $\cos \theta (2 \cos 3\theta - 1) = 0$ $\therefore \cos \theta = 0, \quad \cos 3\theta = \frac{1}{2}$ $\theta = \frac{\pi}{2} + n\pi$ $3\theta = 2n\pi \pm \frac{\pi}{3}$ $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$	M1 A1 A1 m1 A1 (m1) A1 [6]	M0 no working For both Use of cos general solution oe If not awarded previously oe
13.a) i)	METHOD 1: Rearrange first equation: $y = \frac{1}{10} \left(\frac{dx}{dt} - 3x \right)$ Substituting into second equation: $\frac{d}{dt} \left(\frac{1}{10} \left(\frac{dx}{dt} - 3x \right) \right) = x + 6 \left(\frac{1}{10} \left(\frac{dx}{dt} - 3x \right) \right)$ $\frac{1}{10} \frac{d^2x}{dt^2} - \frac{3}{10} \frac{dx}{dt} = x + \frac{6}{10} \frac{dx}{dt} - \frac{18}{10} x$ $\frac{1}{10} \frac{d^2x}{dt^2} - \frac{9}{10} \frac{dx}{dt} + \frac{8}{10} x = 0$ $\frac{d^2x}{dt^2} - 9 \frac{dx}{dt} + 8x = 0$ METHOD 2: Differentiating: $\frac{d^2x}{dt^2} = 3 \frac{dx}{dt} + 10 \frac{dy}{dt}$ $\frac{d^2x}{dt^2} = 3 \frac{dx}{dt} + 10(x + 6y)$ Substituting for y $\frac{d^2x}{dt^2} = 3 \frac{dx}{dt} + 10x + 6 \left(\frac{dx}{dt} - 3x \right)$ $\frac{d^2x}{dt^2} = 3 \frac{dx}{dt} + 10x + 6 \frac{dx}{dt} - 18x$ $\frac{d^2x}{dt^2} - 9 \frac{dx}{dt} + 8x = 0$	M1 M1 A1 A1 (M1) (M1) (A1) (A1) (4)	Convincing or $\frac{d^2x}{dt^2} = 3 \frac{dx}{dt} + 10x + 60y$ Convincing

Qu.	Solution	Mark	Notes
13.a) ii)	The auxiliary equation is: $m^2 - 9m + 8 = 0$ $(m - 1)(m - 8) = 0$ $m = 1$ or $m = 8$ Therefore, the general solution is: $x = Ae^t + Be^{8t}$	B1 B1 B1 (3)	
b)	Differentiating gives: $\frac{dx}{dt} = Ae^t + 8Be^{8t}$ Therefore, $y = \frac{1}{10}((Ae^t + 8Be^{8t}) - 3(Ae^t + Be^{8t}))$ $y = \frac{1}{10}(-2Ae^t + 5Be^{8t})$	M1 A1 m1 A1 (4)	FT their x provided 2 real roots cao
c)	When $t = 0$, $\frac{dx}{dt} = 5$ $\therefore 5 = A + 8B$ Also, $y = 4x$ $\frac{1}{10}(-2Ae^t + 5Be^{8t}) = 4(Ae^t + Be^{8t})$ When $t = 0$, $\frac{1}{10}(-2A + 5B) = 4(A + B)$ $\therefore A = -\frac{35}{42}B \quad \left(= -\frac{5}{6}B \right)$ Also, $5 = -\frac{5}{6}B + 8B$ $B = \frac{30}{43}$ $A = -\frac{25}{43}$ Therefore, the particular solution is: $x = -\frac{25}{43}e^t + \frac{30}{43}e^{8t}$	B1 M1 A1 M1 A1 A1 (6) [17]	Equating y and $4x$ Correct equation Attempt to remove one variable Both values FT A, B above